Topological Persistence and Simplification

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拓扑数据分析

用拓扑的方法来分析数据的结构:

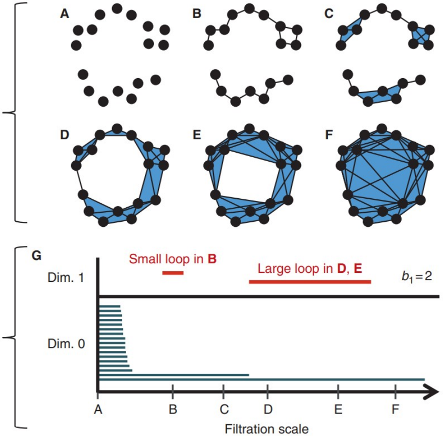

图: An example of TDA

同调群

1. 拓扑空间，奇异单/复形；

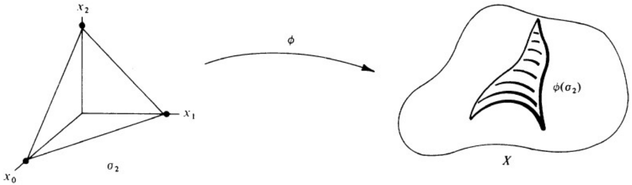

图: 奇异单形

同调群

2. {链复形，链映射}

$$
\begin{tikzcd}
\cdots \arrow[r] & C_{n+1}(X) \arrow[r, "\partial"] \arrow[d, "f_\sharp"] & C_n(X) \arrow[r, "\partial"] \arrow[d, "f_\sharp"] & C_{n-1}(X) \arrow[r] \arrow[d, "f_\sharp"] & \cdots \\
\cdots \arrow[r] & C_{n+1}(Y) \arrow[r, "\partial"] & C_n(Y) \arrow[r, "\partial"] & C_{n-1}(Y) \arrow[r] & \cdots
\end{tikzcd}
$$

图: 链复形与链映射

3. 同调群，Betti 数

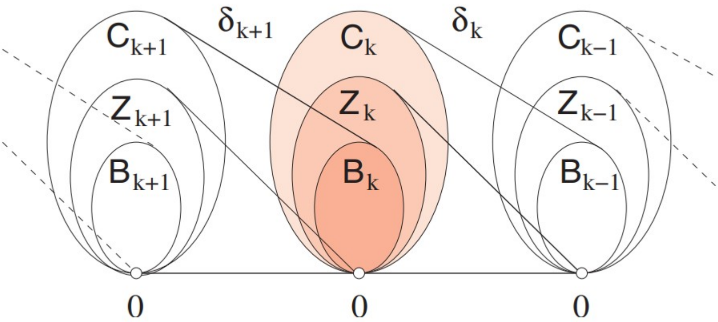

图: Z_k, B_k, H_k

持续同调

1. Persistence 复形;

$$
\begin{tikzcd}
{} \arrow[d, "\partial_3"'] & {} \arrow[d, "\partial_3"'] & {} \arrow[d, "\partial_3"'] & \\
C_2^0 \arrow[r, "f^0"] \arrow[d, "\partial_2"'] & C_2^1 \arrow[r, "f^1"] \arrow[d, "\partial_2"'] & C_2^2 \arrow[r, "f^2"] \arrow[d, "\partial_2"'] & \cdots \\
C_1^0 \arrow[r, "f^0"] \arrow[d, "\partial_1"'] & C_1^1 \arrow[r, "f^1"] \arrow[d, "\partial_1"'] & C_1^2 \arrow[r, "f^2"] \arrow[d, "\partial_1"'] & \cdots \\
C_0^0 \arrow[r, "f^0"] & C_0^1 \arrow[r, "f^1"] & C_0^2 \arrow[r, "f^2"] & \cdots
\end{tikzcd}
$$

图: Persistence 复形

2. Persistence 同调作为分次模的结构;
3. Betti barcode（区间）;

参考文献

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